Investigation of transition radiation from a regular-roughness interface

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(Received 25 January 2001; published 23 July 2001)

Transition radiation at the interface between two media having regular inhomogeneities is considered in the framework of perturbation theory, assuming that the dielectric constants of the two media differ insignificantly from each other. The advantage of this approach is that no limitations exist on the characteristic size of the inhomogeneities. The general case of a surface with two-dimensional roughness is considered. Attention is paid to radiation characteristics that are general for periodic structures, independently of their particular features. The physical picture of the radiation from a rough interface is determined by both longitudinal and transverse effects. The case of normal incidence of an electron on a planar interface having either single or multiple rectangular lugs (diffraction grating) is analyzed in detail. For the single lug infinite in one dimension, simple expressions for transition radiation intensity are obtained, for both relativistic and nonrelativistic electrons. They demonstrate that corrections to the plane-surface intensity become significant at small radiation angles. In this case the radiation is also completely unpolarized, i.e., the spectral energy density of the perpendicular component has the same order of magnitude as that of the parallel component. Consideration of these issues is important since rough surfaces, as compared to planar surfaces, give a number of additional results that may be used for the analysis of surface irregularities.

DOI: 10.1103/PhysRevE.64.026610 PACS number(s): 41.20.−q, 34.50.Dy

I. INTRODUCTION

The well known paper of Ginzburg and Frank [1] devoted to transition radiation is limited to the consideration of an ideally planar interface between two media. In the present paper we consider radiation arising when a charged particle passes through a planar interface on which various inhomogeneities are present.

It is known that a charged particle is not affected by medium inhomogeneities in longitudinal directions if the coherence length exceeds the mean distance between the atoms of the medium or individual inhomogeneities (see, e.g., [2]). Inhomogeneities on a plane interface may be of various types, such as separate inhomogeneities isolated from each other, periodically located inhomogeneities, or statistical ones. These problems have been treated theoretically in [3,4]. The first experiments on transition radiation already included information concerning the influence of the treatment of the interface on the polarization of the transition radiation (see, e.g., [5]). Later, further experimental work appeared devoted to this problem [6].

In addition to the inhomogeneities indicated above, bent interfaces are also possible as well as ideally planar interfaces having an intermediate layer. If, however, the typical parameters determining the interface curvature are small compared to the corresponding typical distances playing a role in the formation of the radiation, corrections to formulas for an ideal interface are not essential [7,8]. Transition radiation on an ideally planar interface having an intermediate layer (indistinct boundary), considered for a number of special cases [9,10], differs only slightly from the case of an ideally distinct interface, if the longitudinal coherence length exceeds the size of the intermediate layer.

To clarify the physical meaning of the stated problem we note that in transition radiation at an ideally planar interface the momentum transferred to the interface is always directed perpendicularly to it during the emission process. If the interface is the plane (x,y) and the particle moves along the z axis at the velocity $\mathbf{v}$, the momentum $\mathbf{q}^{||}$ is transferred to the interface along the direction of motion only. The momentum transferred to the interface can easily be obtained from the laws of conservation of energy,

$$\Delta E=\hbar\omega, \tag{1}$$

and longitudinally transferred momentum.

$$\left(\frac{\Delta\mathbf{p}}{\hbar}-\mathbf{k}\right)^{||}=\mathbf{q}^{||}, \tag{2}$$

where ΔE and $\Delta\mathbf{p}$ are the differences in the energy and momentum of the emitting particle before and after emission of a photon with energy $\hbar\omega$ and wave vector $\mathbf{k}=(\omega/c)\sqrt{\varepsilon_0}\mathbf{n}$ ($\mathbf{n}$ being the unit vector in the $\mathbf{k}$ direction). Using the relation

$$\Delta E=\mathbf{v}\cdot\Delta\mathbf{p}, \tag{3}$$

we obtain, for small changes of the energy and momentum,

$$q^{||}=\frac{\omega}{v}(1-\beta\sqrt{\varepsilon_0}\cos\theta), \tag{4}$$

where θ is the angle between the directions of the emitted photon and particle velocity, and $\beta=v/c$, where c is the velocity of light.

In the presence of inhomogeneities on a planar interface a typical length l appears that characterizes the medium inhomogeneity in the (x,y) plane. In this case the medium can take a momentum of the order of $\sim\hbar/l$ in the transverse

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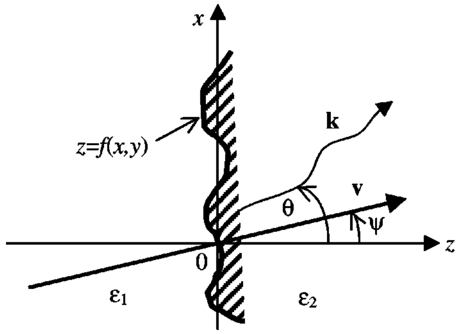

FIG. 1. Section of two-dimensional rough surface in the (x,z) plane.

direction also. Thus, if we represent the field of the particle as expanded in pseudophotons, whose momenta we denote by $\mathbf{k}(k_x', k_y', k_z' = \omega/v)$ the momentum conservation law may be rewritten as

$$k_z' - k_z = q^{||}, \tag{5}$$

$$k_\perp' - k_\perp \approx \frac{1}{l}. \tag{6}$$

The relation (5) is equivalent to Eq. (4) since $k_z' = \omega/v$, and the relation (6) should lead to a change of the Ginzburg-Frank formulas in just the same manner as, in light scattering, a relation similar to Eq. (6), in which the pseudophoton is replaced by the scattered photon, leads to deviations from the Fresnel laws of reflection and refraction. The extent to which this change is significant depends on specific calculations.

As the exact solution of the problem seems to be very complex, we will use perturbation theory to reveal the physical picture.

II. STARTING EXPRESSION FOR THE INTENSITY OF TRANSITION RADIATION FROM AN ARBITRARILY SHAPED INTERFACE

Let us consider radiation arising when a charged particle traverses the interface between two media. Let the interface be given by a function $z = f(x,y)$. This function describes small deviations of the real interface from the plane $z = 0$ that would have been the interface between the two media in the case of ideal surfaces. These deviations are caused by surface roughness. Let us choose the x axis of the Cartesian coordinate system in the plane containing the particle velocity $\mathbf{v}$ and the z axis. The velocity $\mathbf{v}$ is directed from the first medium into the second and makes an angle ψ with the z axis (Fig. 1).

For calculating the radiation we will use light scattering theory (see, e.g., [11]) replacing the electromagnetic wave in it by the field of the moving particle, which we expand in a Fourier integral with respect to time, following the standard procedure. In such a way the emission problem is reduced to that of the scattering of a set of monochromatic waves constituting the field of the moving particle (see [2] for more detail). It should then be assumed that the dielectric constants of the two media differ from each other insignificantly. A more rigorous criterion for the applicability of this approach is given below. Thus, the calculation of transition radiation performed below with the use of perturbation theory is applicable to only a limited set of interfaces between two media with slightly different refractive indices. Such interfaces may be, for example, those between solid particles and the corresponding liquids these particles are immersed in. Although the output of transition radiation is proportional to the square of the difference in refractive indices of the two media and hence is strongly suppressed in the case of slightly differing indices, the calculation technique employed will allow one to look into the physical picture of the phenomenon and to obtain general formulas valid for any of interfaces indicated above. Many of the qualitative conclusions will obviously remain valid also for interfaces between two media with sharply changing optical properties. Moreover, for an approximate quantitative evaluation of the radiation in the case of abruptly changing properties of the media, one may make use of an interpolation formula based on replacing in the final expressions the factor corresponding to the emission from a planar boundary calculated perturbatively by the exact formula for transition radiation from the planar interface.

The radiation energy at large distances R_0 in the frequency range $d\omega$ and into the solid angle interval $d\Omega$ for an arbitrarily shaped interface is determined by the usual expression of classical electrodynamics with allowance for the dielectric constant of the medium:

$$dI(\omega,\mathbf{k}) = c\sqrt{\varepsilon_0}|\mathbf{E}_\omega'|^2 R_0^2\, d\Omega\, d\omega, \tag{7}$$

where $\varepsilon_0 = (\varepsilon_1 + \varepsilon_2)/2$ is the mean value of the dielectric constant of the two media. We denote by $\mathbf{E}_\omega'$ the radiation field strength at the frequency ω at large distances from the interface, it is determined from the macroscopic Maxwell's equations [2],

$$\mathbf{E}_\omega' = -\frac{e^{ikR_0}}{4\pi R_0}\left[\mathbf{k}\left[\mathbf{k}, \int_{-\infty}^{\infty} \mathbf{E}_\omega(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} \varepsilon'(\mathbf{r}) dV_r\right]\right], \tag{8}$$

where $\mathbf{E}_\omega(\mathbf{r})$ is the Fourier component of the field of a uniformly moving particle at the point $\mathbf{r}(x,y,z)$ in the medium with mean dielectric constant ε_0 [2],

$$\mathbf{E}_\omega(\mathbf{r}) = \frac{ie}{2\pi^2 v_z}\int_{-\infty}^{\infty} \frac{\omega\mathbf{v}/c^2 - \mathbf{k}/\varepsilon_0}{k^2 - (\omega^2/c^2)\varepsilon_0} \times e^{ik_x'x + ik_y'y + ik_z'z}\, dk_x\, dk_y, \tag{9}$$

with e being the electron charge. The quantity $\varepsilon'(\mathbf{r})$ is the deviation of the dielectric constant from ε_0, e.g., in our case

$$\varepsilon'(\mathbf{r}) = \begin{cases} \varepsilon_1 - \varepsilon_0 = \dfrac{\varepsilon_1 - \varepsilon_2}{2}, & -\infty < z < f(x,y) \\ \varepsilon_2 - \varepsilon_0 = \dfrac{\varepsilon_2 - \varepsilon_1}{2}, & f(x,y) < z < +\infty. \end{cases} \tag{10}$$

As the mean value of $\varepsilon'(\mathbf{r})$ is equal to zero, the mean value of the scattered field vanishes as well. By integrating expression (8) over z and using Eq. (10) we obtain

$$\mathbf{E}'_\omega=\frac{e(\varepsilon_2-\varepsilon_1)}{8\pi^3 v_z\varepsilon_0}\frac{e^{ikR_0}}{R_0}\times\left[\mathbf{k}\left[\mathbf{k},\int_{-\infty}^{\infty}\frac{\omega\mathbf{v}/c^2-\mathbf{k}'/\varepsilon_0}{q^{\|}[k'^2-(\omega^2/c^2)\varepsilon_0]}e^{iq^{\|}f(x,y)}\times\exp\{i(k'_x-k_x)x+i(k'_y-k_y)y\}dk'_x\,dk'_y\,dx\,dy\right]\right], \tag{11}$$

where

$$\omega=\mathbf{k}'\cdot\mathbf{v}=k'_x v_x+k'_z v_z, \tag{12}$$

$$q^{\|}=k'_z-k_z=\frac{\omega-k'_x v_x}{v_z}-k_z.$$

In what follows let $v_x=0,\ v_z=v$. When $z=f(x,y)=0$, i.e., in the case of a planar interface, in the approximation of perturbation theory, we can perform the integration over the remaining variables and obtain a formula for the transition radiation in the cases of one and two interfaces [3]. Comparison with the exact formulas for the transition radiation allows us to find the condition of applicability of the perturbation theory method. It is easy to show that in addition to the condition

$$\left|\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}\right|\ll 1 \tag{13}$$

a stricter condition should be satisfied, that is,

$$\left|\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}\right|\ll\frac{\lambda}{l_{coh}}\cos\theta, \tag{14}$$

where $2\pi\lambda$ is the wavelength of the emitted photon, and θ is the angle of the emission measured from the z axis; $0\leqslant\theta\leqslant\pi$. The coherence length

$$l_{coh}=\frac{1}{q^{\|}}=\frac{\lambda\beta\sqrt{\varepsilon_0}}{1-\beta\sqrt{\varepsilon_0}\cos\theta} \tag{15}$$

is defined here as the inverse of the momentum transferred longitudinally to the interface in the emission of a photon in the direction θ [Eq. (4)]; this corresponds, in classical considerations, to that length of the trajectory of the emitting particle that plays a role in the formation of transition radiation (see, e.g., [2]).

For nonrelativistic particles $l_{coh}\sim\lambda\beta\sqrt{\varepsilon_0}$ and the condition (14) for $\theta\neq\pi/2$ is actually always weaker than (13), while for relativistic particles the condition (14) can be noticeably stronger than (13).

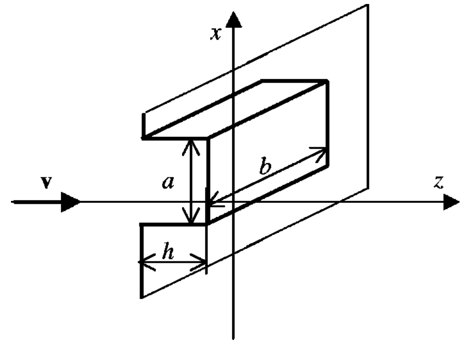

FIG. 2. Rectangular-lug-shaped inhomogeneity.

III. RADIATION FROM AN IDEAL INTERFACE HAVING AN ISOLATED RECTANGULAR-LUG-SHAPED INHOMOGENEITY

As an example yielding to analytical solution let us consider emission of a charged particle on an interface given by the following expression (Fig. 2):

$$z=f(x,y)=\begin{cases} h, & |x-x_0|\leqslant\frac{a}{2},\quad |y-y_0|\leqslant\frac{b}{2}\\ 0, & |x-x_0|>\frac{a}{2},\quad |y-y_0|>\frac{b}{2},\end{cases} \tag{16}$$

where (x_0,y_0) is the distance between the trajectory of the particle moving along the z axis and the center of the inhomogeneity located in the plane $z=0$; h is the height while a and b are the widths of the roughness along the x and y axes, respectively.

Simple integration of formula (11) with regard to Eq. (16) leads to the following expression for the transition radiation field at large distances R_0 with a particle moving along the z axis:

$$\mathbf{E}'_\omega=\mathbf{E}^{pl}_\omega+\frac{e(\varepsilon_2-\varepsilon_1)}{2\pi^3 v\varepsilon_0}l_{coh}\frac{e^{ikR_0}}{R_0}(e^{ih/l_{coh}}-1)\times\int_{-\infty}^{\infty}\left[\mathbf{k}\left[\mathbf{k},\frac{\omega\mathbf{v}/c^2-\mathbf{k}'/\varepsilon_0}{k'^2-(\omega^2/c^2)\,\varepsilon_0}\right]\right]\times\frac{\sin(\Delta k_x a/2)}{\Delta k_x}\frac{\sin(\Delta k_y b/2)}{\Delta k_y}e^{i\Delta k_x x_0+i\Delta k_y y_0}dk'_x\,dk'_y, \tag{17}$$

where

$$\Delta k_x=k'_x-k_x, \tag{18}$$

$$\Delta k_y=k'_y-k_y$$

is the momentum transferred to the interface in the transverse direction. In Eq. (17) we singled out the field of the transition radiation from the plane interface $\mathbf{E}^{pl}_\omega$, i.e., for $h=0$,

$$E_\omega^{pl}=\frac{e(\varepsilon_2-\varepsilon_1)}{2\pi c\varepsilon_0}\beta\sin\theta\frac{e^{ikR_0}}{R_0}\times\frac{1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta}{(1-\beta^2\varepsilon_0\cos^2\theta)(1-\beta\sqrt{\varepsilon_0}\cos\theta)}. \qquad (19)$$

The polarization of the radiation is determined by the double vector product under the integral sign in formula (17). If the particle moves along the z axis, two polarizations are possible in our case in contrast to that of the planar interface. In addition to parallel polarization ($||$) with the electric vector lying in the radiation plane (containing the wave vector of the emitted quantum, $\mathbf{k}$, and the normal to the plane $z=0$), a perpendicular polarization ($\perp$) appears with the electric vector perpendicular to the radiation plane. This follows from the relation

$$\left[\mathbf{k}\left[\mathbf{k},\left(\frac{\omega\mathbf{v}}{c^2}-\frac{\mathbf{k}'}{\varepsilon_0}\right)\right]\right]=\mathbf{n}_{||}\frac{k}{\varepsilon_0}\left[(k_xk_x'+k_yk_y')\frac{k_z}{k_\rho}-k_\rho\frac{\omega}{v}(1-\beta^2\varepsilon_0)\right]+\mathbf{n}_\perp\frac{k^2}{k_\rho\varepsilon_0}(k_xk_y'-k_yk_x'), \qquad (20)$$

$$k_\rho^2=k_x^2+k_y^2,$$

where $\mathbf{n}_{||},\mathbf{n}_\perp$ are unit vectors in the radiation plane and in the plane perpendicular to that, respectively. It is seen from Eq. (20) that if $k_x'=k_x, k_y'=k_y$ we have $(\mathbf{E}_\omega')^\perp=0$.

Expression (17) shows that the particle does not experience the inhomogeneity of the interface at transversely transferred momenta large compared to the inhomogeneity "momentum" $\hbar/a$ or $\hbar/b$, if the conditions

$$\left|\frac{\Delta k_x}{2}a\right|\gg1,\quad\left|\frac{\Delta k_y}{2}b\right|\gg1,\quad \text{i.e.,}\quad a\gg\rho_{eff},\quad b\gg\rho_{eff}, \qquad (21)$$

are satisfied, since the effective k_x' and k_y' are determined from the denominator of the integrand, $k'^2_x+k'^2_y\leqslant(\omega^2/v^2)/(1-\beta^2\varepsilon_0)=\rho_{eff}^{-1}$. In this case the rapidly oscillating factors may be replaced by δ functions $\delta(\Delta k_x)\delta(\Delta k_y)$, resulting in the law of conservation of transversely transferred momentum in the emission process: $k_x'=k_x$, $k_y'=k_y$. By integrating expression (17) over k_x',k_y' we arrive at the formula (19) with an additional phase shift $\exp(ih/l_{coh})$. This corresponds to the laws of conservation of the transverse components of momenta in the reflection and refraction of light at a planar boundary. Both of these conservation laws result from the fact that in the case of a perfectly planar boundary the surface cannot accept any momentum along the interface. However, if inhomogeneities are present on the interface between two media, the conservation laws $k_x'=k_x$, $k_y'=k_y$ may be violated. This will lead to a difference of the formulas for transition radiation on a rough interface from those on a planar one.

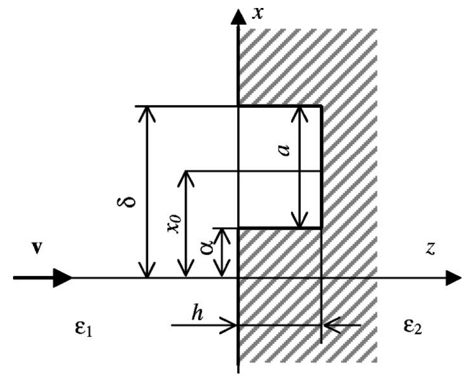

FIG. 3. Section of strip in the (x,z) plane.

So, in the process of transition radiation the particle is affected by an inhomogeneity only at momentum transfers smaller than or of the order of $\hbar/a$ and $\hbar/b$. At $a=0$ and $b=0$ the inhomogeneity disappears and the effect of surface inhomogeneity vanishes.

In just the same manner the particle does not experience inhomogeneities when

$$h\ll l_{coh} \qquad (22)$$

for arbitrary values of a and b. The last fact is well known and related to the concept of the coherence length along the direction of the particle's motion [2].

The analysis of formula (17) will be performed in the limit $b\to\infty$, i.e., when the rectangular lug becomes a strip-shaped inhomogeneity (Fig. 3). In this case it is possible to integrate expression (17) completely.

For spectral densities of the radiation energy $I=dI(\omega,\mathbf{k})/(d\Omega\, d\omega)$, in the case of a rectangular strip-shaped lug ($b\to\infty$), in the absence of Cherenkov-Vavilov radiation ($\beta\sqrt{\varepsilon_0}\leqslant1$) we obtain the following expressions for, respectively, parallel and perpendicular polarizations:

$$I^{||}=I_{pl}^{||}\left\{1+(v_\alpha-v_\delta)\sin\frac{h}{l_{coh}}+[2(\operatorname{sgn}\alpha-\operatorname{sgn}\delta)+2(1-\operatorname{sgn}\alpha\operatorname{sgn}\delta)(1+\mu_\alpha+\mu_\delta)+(\mu_\alpha\operatorname{sgn}\alpha-\mu_\delta\operatorname{sgn}\delta)(2+\mu_\alpha\operatorname{sgn}\alpha-\mu_\delta\operatorname{sgn}\delta)+(v_\alpha-v_\delta)^2]\sin^2\frac{h}{2l_{coh}}\right\}, \qquad (23)$$

$$I^\perp=I_{pl}^{||}A[e^{-2|\alpha|/\rho}+e^{-2|\delta|/\rho}-2e^{-(|\alpha|+|\delta|)/\rho}\cos(ak_x)]\sin^2\frac{h}{2l_{coh}}.$$

Here $I_{pl}^{||}$ is the spectral density of radiation energy determined by expressions (7) and (19); $\alpha=x_0-a/2$, $\delta=x_0+a/2$ are the distances of the particle trajectory from the boundaries of the inhomogeneity; φ is the azimuthal angle of the emitted photon measured from the x axis, $0\leqslant\varphi\leqslant2\pi$;

$$\mu_{\alpha,\delta}=[F\sin(|\alpha,\delta|k_x)-\cos(|\alpha,\delta|k_x)]e^{-|\alpha,\delta|/\rho};$$

$$v_{\alpha,\delta}=[F\cos(|\alpha,\delta|k_x)+\sin(|\alpha,\delta|k_x)]e^{-|\alpha,\delta|/\rho};$$

$$F=\frac{\rho}{\lambda\beta\sqrt{\varepsilon_0}}\,\frac{(1-\beta^2\varepsilon_0)(\cos\theta+\beta\sqrt{\varepsilon_0}\sin^2\theta)\cos\varphi}{(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)\sin\theta};\tag{24}$$

$$A=\frac{\rho^2}{\lambda^2\beta^2\varepsilon_0}\,\frac{(1-\beta^2\varepsilon_0\cos^2\theta)^2\sin^2\varphi}{(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)^2\sin^2\theta};$$

$$\rho=\frac{\lambda\beta\sqrt{\varepsilon_0}}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}};\quad k_x=\frac{\sin\theta\cos\varphi}{\lambda}.$$

Consider some special cases of expressions (23). For $h \ll l_{coh}$ and arbitrary a additions to the transition radiation from a planar interface are proportional to h/l_{coh} in the case of parallel polarization, and to $(h/l_{coh})^2$ in the case of perpendicular polarization; for $h\to 0$ we have $I^{||}\to I^{||}_{pl}$, $I^{\perp}\to 0$.

Expressions (23) pass to the formulas for a planar interface at $a=0$ or at $a\to\infty$, i.e., in the absence of inhomogeneities. The intensity of the radiation depends on the relative signs of α and δ. When the particle moves outside the inhomogeneity, sgn α and sgn δ are the same, while when moving inside sgn α and sgn δ are different.

When the conditions

$$|\alpha|\gg\rho,\quad |\delta|\gg\rho,\tag{25}$$

are simultaneously satisfied, formulas (23) and (24) show that the corrections to radiation at a planar interface are exponentially small. This result may be explained by the "limited nature" of the transverse sizes of the field of the moving particle. If we expand the electric field of a rapidly moving particle in a Fourier integral with respect to time, the spectral density of the particle's field will turn out to range up to the frequency ω only for impact parameters (distance from the point at which the particle's field is being considered to the trajectory in the direction perpendicular to the particle's motion) shorter than ρ_{eff},

$$\rho\leqslant\rho_{eff}=\frac{\lambda\beta\sqrt{\varepsilon_0}}{\sqrt{1-\beta^2\varepsilon_0}}.\tag{26}$$

At larger impact parameters the particle's field spectrum does not contain photons at frequencies exceeding ω (see, e.g., [2]). In the case under study the distances from the particle's trajectory to both edges of the rectangular strip considerably exceed the lateral sizes of the field, since $\rho\leqslant\rho_{eff}$. Due to this fact the corrections to the transition radiation at a perfectly planar boundary are exponentially small.

In the other limiting case where conditions opposite to those in Eq. (25),

$$|\alpha|\ll\rho,\quad |\delta|\ll\rho,\tag{27}$$

are satisfied, i.e., when the transverse field of the particle is actually uniform on the whole strip, the corrections to radiation at a planar boundary may readily be obtained from the formula (23) above after expanding in the parameters $|\alpha|/\rho$ and $|\delta|/\rho$.

Owing to the fact that the relation

$$\frac{\beta\sqrt{\varepsilon_0}\sin\theta\cos\varphi}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}}\leqslant 1\tag{28}$$

is almost always valid, from the inequality (27) it follows that

$$\frac{|\alpha,\delta|}{\lambda}\sin\theta\cos\varphi\ll 1.\tag{29}$$

Making use of Eq. (29), under conditions (27), we obtain from Eq. (23)

$$\begin{aligned}I^{||}=I^{||}_{pl}\Bigg[1-&\frac{|\alpha|-|\delta|}{\rho}\\ &\times\left(F-\frac{\beta\sqrt{\varepsilon_0}\sin\theta\cos\varphi}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}}\right)\sin\frac{h}{l_{coh}}\\ &-2\frac{a}{\rho}\left(1+F\frac{\beta\sqrt{\varepsilon_0}\sin\theta\cos\varphi}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}}\right)\sin^2\frac{h}{2l_{coh}}\Bigg],\end{aligned}\tag{30}$$

$$\begin{aligned}I^{\perp}=I^{||}_{pl}A\Bigg[&\left(\frac{|\alpha|-|\delta|}{\rho}\right)^2\\ &+\frac{a^2}{\rho^2}\frac{\beta^2\varepsilon_0\sin^2\theta\cos^2\varphi}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}}\Bigg]\sin^2\frac{h}{2l_{coh}}.\end{aligned}$$

Since the corrections should vanish for $a\to 0$, destructive interference takes place from different edges of the strip. For nonrelativistic particles ($\beta\sqrt{\varepsilon_0}\ll 1$, $\rho\sim\rho_{eff}\sim l_{coh}\sim\lambda\beta\sqrt{\varepsilon_0}$) the expressions for F and A have the forms

$$F\approx\cot\theta\cos\varphi,\quad A\approx\frac{\sin^2\varphi}{\sin^2\theta}.\tag{31}$$

For relativistic particles ($\beta\sqrt{\varepsilon_0}\sim 1$, $\theta\ll 1$) we have

$$F\approx\frac{\sqrt{1-\beta^2\varepsilon_0}\cos\varphi}{\theta\sqrt{1+\theta^2\sin^2\varphi/(1-\beta^2\varepsilon_0)}},$$

$$A\approx\frac{(1-\beta^2\varepsilon_0)[1+\theta^2/(1-\beta^2\varepsilon_0)]^2\sin^2\varphi}{\theta^2[1+\theta^2\sin^2\varphi/(1-\beta^2\varepsilon_0)]}.\tag{32}$$

In both cases maximal deviations from the formulas of plane-interface transition radiation are observed for small angles of radiation: $\theta\ll 1$ if $\beta\sqrt{\varepsilon_0}\ll 1$ and $\theta\ll\sqrt{1-\beta^2\varepsilon_0}$ if $\beta\sqrt{\varepsilon_0}\sim 1$.

Significant changes in the formulas for transition radiation at a planar boundary will be observed if one of the conditions (27) is satisfied, while instead of the other one the opposite condition holds:

$$\alpha| \ll \rho, \quad |\delta| \gg \rho, \tag{33}$$

i.e., if the particle is affected by only the nearest edge α. In this case interference of radiation from the different edges of the strip does not occur and one may expect significant effects. For spectral densities of the radiation energy we obtain

$$I^{||} = I^{||}_{pl}\Bigg\{1 + \Bigg[F\cos\left(\frac{|\alpha|}{\lambda}\sin\theta\cos\varphi\right) + \sin\left(\frac{|\alpha|}{\lambda}\sin\theta\cos\varphi\right)\Bigg]\sin\frac{h}{l_{coh}} + \Bigg[1 + F^2 + 2\Bigg(F\sin\left(\frac{|\alpha|}{\lambda}\sin\theta\cos\varphi\right) - \cos\left(\frac{|\alpha|}{\lambda}\sin\theta\cos\varphi\right)\Bigg)\Bigg]\sin^2\frac{h}{2l_{coh}}\Bigg\}, \tag{34}$$

$$I^{\perp} = I^{||}_{pl}A\sin^2\frac{h}{2l_{coh}}.$$

For nonrelativistic particles formulas (34) simplify to

$$I^{||} = I^{||}_{pl}\Bigg[1 + \cot\theta\cos\varphi\sin\frac{h}{l_{coh}} + (\cot^2\theta\cos^2\varphi - 1)\sin^2\frac{h}{2l_{coh}}\Bigg], \tag{35}$$

$$I^{\perp} = I^{||}_{pl}\frac{\sin^2\varphi}{\sin^2\theta}\sin^2\frac{h}{2l_{coh}}.$$

In this case the spectral densities of the radiation energy for parallel and perpendicular polarizations for $\theta \ll 1$ become of the same order, i.e., total depolarization of radiation takes place. The absolute values of the intensities in both polarizations for $\theta \ll 1$ can exceed the intensity from the plane interface. After integration over the angles of emission of a photon θ, φ, the overall intensity $I^{||} + I^{\perp}$ becomes equal to the radiation intensity from the planar interface. Hence, the presence of a strip-shaped inhomogeneity results in an angular redistribution of the intensity of plane-interface transition radiation and in its depolarization.

For relativistic particles, under the condition (28), we have

$$I^{||} = I^{||}_{pl}\Bigg[1 + F\sin\frac{h}{l_{coh}} + (F^2 - 1)\sin^2\frac{h}{2l_{coh}}\Bigg], \tag{36}$$

$$I^{\perp} = I^{||}_{pl}A\sin^2\frac{h}{2l_{coh}},$$

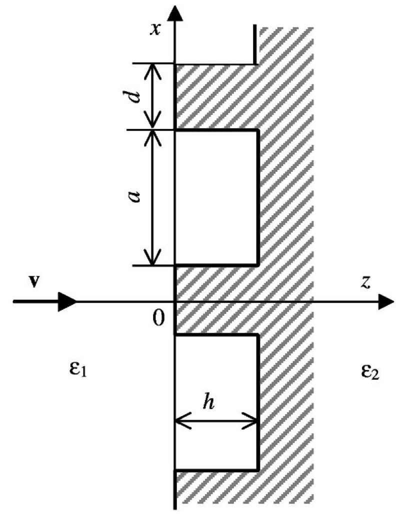

FIG. 4. Section of two-dimensional grating in the (x,z) plane.

where $l_{coh} \sim \lambda/2$ for backward emission and $l_{coh} \sim \lambda/(1 - \beta^2\varepsilon_0)$ for forward emission. For $\theta \ll \sqrt{1 - \beta^2\varepsilon_0}$ we obtain from Eqs. (32)

$$F \approx -\frac{\cos\varphi}{\theta}\sqrt{1 - \beta^2\varepsilon_0}, \tag{37}$$

$$A \approx \frac{\sin^2\varphi}{\theta^2}(1 - \beta^2\varepsilon_0).$$

Hence, in the relativistic case also there is, for $\theta \ll \sqrt{1 - \beta^2\varepsilon_0}$, a considerable deviation of the intensity from that in the plane-interface case. But at angles $\sqrt{1 - \beta^2\varepsilon_0} \ll \theta \ll 1$ these deviations are significantly smaller.

It follows from the analysis performed that the corrections are significant at small angles of emission where total depolarization occurs. The maximal effect is achieved when the sizes of the inhomogeneity in the (x,y) plane are of the order of or larger than the transverse sizes of the field of a particle that moves near one of the inhomogeneity (step) edges.

IV. TRANSITION RADIATION FROM DIFFRACTION GRATING

Let us consider an interface in the form of a set of periodically placed lugs having widths a and b and separated from each other by distances d and g along the x and y axes, respectively (diffraction grating, Fig. 4):

$$z = f(x,y) = \begin{cases} h; & |x - x_m| \leq \dfrac{a}{2}, \quad |y - y_s| \leq \dfrac{b}{2} \\ 0; & |x - x_m| > \dfrac{a}{2}, \quad |y - y_s| > \dfrac{b}{2}, \end{cases} \tag{38}$$

where x_m and y_s are the distances from the particle trajectory to the center of the inhomogeneity with indices m and s,

$$x_m = x_0 + m l_x, \quad y_s = y_0 + s l_y, \tag{39}$$
$$m,s = 0, \pm 1, \pm 2, \ldots .$$

with $l_x = a + d$, $l_y = b + g$ being the periods along the x and y axes, respectively. Substituting Eq. (38) into Eq. (11) and taking into account that

$$\int_{-\infty}^{\infty} \cdots dx \cdots \int_{-\infty}^{\infty} \ldots dy$$
$$= \sum_{m,s=-\infty}^{\infty} \int_{x_m - a/2}^{x_m + a/2} \ldots dx \int_{y_s - b/2}^{y_s + b/2} \cdots dy, \tag{40}$$

we obtain for the radiation field in the case of a finite grating $(-N \leqslant m,\ s \leqslant N)$ with $(2N+1)^2$ lugs the following expression:

$$\mathbf{E}'_\omega = \mathbf{E}^{pl}_\omega + \int_{-\infty}^{\infty} \mathbf{J}(k_x, k'_y) \frac{\sin[(2N+1)\Delta k_x l_x/2]}{\sin(\Delta k_x l_x/2)} \times \frac{\sin[(2N+1)\Delta k_y l_y/2]}{\sin(\Delta k_y l_y/2)} dk'_x\, dk'_y,$$

$$\mathbf{J}(k'_x, k'_y) = \frac{e(\varepsilon_2 - \varepsilon_1)}{2\pi^3 v \varepsilon_0} l_{coh} \frac{e^{ikR_0}}{R_0} (e^{ih/l_{coh}} - 1) \times \left[\mathbf{k}\left[\mathbf{k}, \frac{\omega \mathbf{v}/c^2 - \mathbf{k}'/\varepsilon_0}{k'^2 - (\omega^2/c^2)\varepsilon_0}\right]\right] \times \frac{\sin(\Delta k_x a/2)}{\Delta k_x} \frac{\sin(\Delta k_y b/2)}{\Delta k_y} e^{i\Delta k_x x_0 + i\Delta k_y y_0}. \tag{41}$$

The structure of the expression (41) yields to an obvious interpretation. For $N=0$ we obtain expression (17) for the case of a single two-dimensional lug. For $N \neq 0$ expression (41) differs from the single-lug case by N -containing factors.

For N sufficiently large, $N \gg 1$, the rapidly oscillating factor may be replaced by a sum of δ functions according to the formula

$$\lim_{N\to\infty} \frac{\sin 2Nx}{\sin x} = \pi \sum_{n=-\infty}^{\infty} \delta(x - n\pi), \tag{42}$$

where n are integers and zero. The substitution (42) may be performed in cases where the remaining part of the integrand $\mathbf{J}(k'_x, k'_y)$ does not vary significantly over the width of the maxima of the function $\sin 2Nx/\sin x$. By integrating we obtain

$$\mathbf{E}'_\omega = \mathbf{E}^{pl}_\omega - \frac{e(\varepsilon_2 - \varepsilon_1)}{8\pi^3 v \varepsilon_0} l_{coh}(e^{ih/l_{coh}} - 1) \frac{e^{ikR_0}}{R_0} \times \sum_{m,s=-\infty}^{\infty} \frac{\left[\mathbf{k}\left[\mathbf{k}, \frac{\omega \mathbf{v}}{c^2} - \frac{\mathbf{k}'}{\varepsilon_0}\right]\right]}{\left(k'^2 - \frac{\omega^2}{c^2}\varepsilon_0\right) ms} \times (e^{i(2\pi m/l_x)(x_0 + a/2)} - e^{i(2\pi m/l_x)(x_0 - a/2)})(e^{i(2\pi s/l_y)(y_0 + b/2)} - e^{i(2\pi s/l_y)(y_0 - b/2)}), \tag{43}$$

where

$$k'_x = k_x + \frac{2\pi}{l_x} m, \tag{44}$$
$$k'_y = k_y + \frac{2\pi}{l_y} s,$$

determines the law of conservation of momentum transferred to the grating in the direction perpendicular to the direction of motion.

Since in a real experiment we deal always with a beam of charged particles, it pays to write down expressions for radiation intensities averaged over the impact parameters $-l_x/2 \leqslant x_0 \leqslant l_x/2$, $-l_y/2 \leqslant y_0 \leqslant l_y/2$. For the spectral densities of the radiation energy averaged in this manner we obtain

$$\overline{I^{\|}} = I^{\|}_{pl}\left(1 - 4\left(\sin^2 \frac{h}{2l_{coh}}\right)\left\{\frac{ab}{l_x l_y} - \sum_{m,s=-\infty}^{\infty} B^2_{ms}\left[1 - \frac{\sqrt{1-\beta^2\varepsilon_0}\cot\theta}{1 - \beta^2\varepsilon_0 - \beta\sqrt{\varepsilon_0}\cos\theta}\left(\frac{2\rho_{eff}}{l_x}\pi m \cos\varphi + \frac{2\rho_{eff}}{l_y}\pi s \sin\varphi\right)\right]^2\right\}\right),$$

$$\overline{I^{\perp}} = I^{\|}_{pl} 4\left(\sin^2 \frac{h}{2l_{coh}}\right) \sum_{m,s=-\infty}^{\infty} B^2_{ms} \frac{(1-\beta^2\varepsilon_0)[(2\rho_{eff}/l_y)\pi s \cos\varphi - (2\rho_{eff}/l_x)\pi m \sin\varphi]^2}{(1 - \beta^2\varepsilon_0 - \beta\sqrt{\varepsilon_0}\cos\theta)^2 \sin^2\theta}, \tag{45}$$

where

$$B_{ms}=\frac{1-\beta^2\varepsilon_0\cos^2\theta}{1-\beta^2\varepsilon_0}\frac{\sin(\pi ma/l_x)}{\pi m}\frac{\sin(\pi sb/l_y)}{\pi s}\times\left[1+\left(\frac{2\rho_{eff}}{l_x}\right)^2\left(\pi m+\frac{\beta\sqrt{\varepsilon_0}\sin\theta\cos\varphi}{\sqrt{1-\beta^2\varepsilon_0}}\frac{l_x}{2\rho_{eff}}\right)^2+\left(\frac{2\rho_{eff}}{l_y}\right)^2\left(\pi s+\frac{\beta\sqrt{\varepsilon_0}\sin\theta\sin\varphi}{\sqrt{1-\beta^2\varepsilon_0}}\frac{l_y}{2\rho_{eff}}\right)^2\right]^{-1}.$$

If the relative variation per step ($m\to m+1,\ s\to s+1$) of the functions to be summed is small, which is the case under the condition $2\rho_{eff}/l_x, l_y \ll 1$, we can replace the sum by an integral making use of Eqs. (44). In this case the field of a particle covers no more than one lug, and we should naturally arrive at formulas for the intensity of radiation on a planar boundary with a single lug. In fact, if we use formula (7) for the fields (17), then, after averaging over a beam having sizes l_x and l_y along the corresponding directions, we obtain Eqs. (45) where instead of sums there are integrals over k'_x, k'_y with integration limits from $-\infty$ to $+\infty$.

If, however, the particle's field covers many lugs on the diffraction grating ($2\rho_{eff}/l_x, l_y \gg 1$) we obtain

$$\overline{I^{||}}\approx I_{pl}^{||}\left[1-4\frac{ab}{l_xl_y}\left(1-\frac{ab}{l_xl_y}\right)\sin^2\frac{h}{2l_{coh}}\right],$$

$$\overline{I^{\perp}}\ll\overline{I^{||}}, \qquad (46)$$

since for m, $s\gg 1$ the omitted terms in the sum over m and s contribute slightly.

In the case of a diffraction grating in the form of a set of rectangular-strip-shaped lugs ($b\to\infty$) the formulas (43) are relatively simple, and for radiation fields with, respectively, parallel and perpendicular polarizations we obtain

$$(E'_\omega)^{||}=E_\omega^{pl}\left\{1-\sum_{m=-\infty}^{\infty}B_m\left[1-\frac{2\rho}{l_x}\pi m\times\frac{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}\cos\theta\cos\varphi}{(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)\sin\theta}\right]\right\},$$

$$(E'_\omega)^{\perp}=E_\omega^{pl}\sum_{m=-\infty}^{\infty}B_m\frac{2\rho}{l_x}\pi m\times\frac{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}\sin\varphi}{(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)\sin\theta}, \qquad (47)$$

$$B_m=-\frac{(1-\beta^2\varepsilon_0\cos^2\theta)(e^{ih/l_{coh}}-1)}{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}\times e^{i(2\pi m/l_x)x_0}\frac{\sin(\pi ma/l_x)}{\pi m}\times\left[1+\left(\frac{2\rho}{l_x}\right)^2\times\left(\pi m+\frac{\beta\sqrt{\varepsilon_0}\sin\theta\cos\varphi}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}}\frac{l_x}{2\rho}\right)^2\right]^{-1}.$$

If on changing each of the integers m by unity the sum varies by a negligibly small value, we can consider m as a continuous variable and hence replace the sum by an integral with use of the relation (44). This can be done in the limiting case of small ρ, $2\rho/l_x\ll 1$ (the particle's field covers no more than one strip on the grating), under conditions (28) and

$$\frac{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}\cos\theta\cos\varphi}{(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)\sin\theta}\ll 1, \qquad (48)$$

and hence we obtain the results (23) for a single strip.

Expressions (47) can easily be calculated in the limiting case of large ρ, $2\rho/l_x\gg 1$ (the particle's field covers $\sim 2\rho/l_x$ inhomogeneities on the diffraction grating) under conditions (28) and

$$\frac{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}\cos\theta\cos\varphi}{(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)\sin\theta}\gg 1. \qquad (49)$$

In this case the spectral densities of the radiation intensity are

$$I^{||}=I_{pl}^{||}\left[1-\frac{|\alpha|-|\delta|}{\rho}\frac{(1-\beta^2\varepsilon_0\cos^2\theta)\cos\theta\cos\varphi}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)\sin\theta}\sin\frac{h}{l_{coh}}+4\frac{a}{l_x}\left(\frac{a}{l_x}-1\right)\sin^2\frac{h}{2l_{coh}}\right], \qquad (50)$$

$$I^{\perp}=I_{pl}^{||}A\left(\frac{|\alpha|-|\delta|}{\rho}\right)^2\sin^2\frac{h}{2l_{coh}}$$

with $0\leqslant 2\pi|\alpha,\delta l|l_x$.

V. TRANSITION RADIATION FROM AN ARBITRARILY PERIODIC SURFACE AT AN ARBITRARY ANGLE OF INCIDENCE OF THE PARTICLE

Let $z=f(x,y)$ be a function periodic with respect to both variables x and y with the periods l_x and l_y, respectively. Then $\exp\{i[(\omega-k_x'v_x/v_z)-k_z]f(x,y)\}$ is also a periodic function. Denoting by L_{ms} the coefficients in the expansion of this function in a double Fourier series

$$\exp\left[i\left(\frac{\omega-k_x'v_x}{v_z}-k_z\right)f(x,y)\right]=\sum_{m,s=-\infty}^{\infty}L_{ms}\exp\{i[(2\pi m/l_x)x+(2\pi s/l_y)y]\},\quad(51)$$

substituting the expansion into Eq. (11), and integrating, we obtain

$$\mathbf{E}'_\omega=\frac{e(\varepsilon_2-\varepsilon_1)}{8\pi^3v_z\varepsilon_0}\frac{e^{ikR_0}}{R_0}\times\sum_{m,s=-\infty}^{\infty}\frac{[\mathbf{k}[\mathbf{k},\omega\mathbf{v}/c^2-\mathbf{k}'/\varepsilon_0]]L_{ms}}{[k'^2-(\omega^2/c^2)\varepsilon_0][(\omega-k_x'v_x)/v_z-k_z)]},\quad(52)$$

where

$$k_x'=k_x-\frac{2\pi}{l_x}m,\quad(53)$$

$$k_y'=k_y-\frac{2\pi}{l_y}s.$$

Comparison with the formulas for a planar boundary calculated with use of perturbation theory shows that at $L_{ms}=1$, $m=s=0$ we obtain these formulas. In fact, if the inhomogeneity tends to the plane, i.e., $f(x,y)\to0$ or $l_x,l_y\to\infty$, we see from Eq. (51) that $L_{ms}\to0$ provided only one of the integers m and s differs from zero, while $L_{00}\to1$, i.e., we obtain the case of emission on the planar boundary. Thus, the problem is reduced to determination of the coefficients L_{ms} in Eq. (51), which describe the shape of the surface. In the case of one-dimensional roughness the calculations are simplified significantly, as the double series in Eq. (51) is reduced to a single series.

Let us give several examples of the expansion coefficients L_{ms}. For an interface sinusoidal in both directions,

$$f(x,y)=a\cos\left(\frac{2\pi}{l_x}x\right)+b\cos\left(\frac{2\pi}{l_y}y\right),\quad(54)$$

we have the following expression for L_{ms}:

$$L_{ms}=i^{m+s}J_m\left(\frac{a}{l_{coh}}\right)J_s\left(\frac{b}{l_{coh}}\right),\quad(55)$$

where $J_m(\chi)$ is a Bessel function of order m,

$$\chi=\frac{a}{l_{coh}},\quad l_{coh}=\left(\frac{\omega-k_x'v_x}{v_z}-k_z\right)^{-1}.\quad(56)$$

In the case of one-dimensional roughness, i.e.,

$$f(x,y)\to f(x)=a\cos\left(\frac{2\pi}{l_x}x\right),\quad(57)$$

we obtain

$$L_{ms}\to L_m=i^mJ_m(\chi).\quad(58)$$

If we superimpose on the regular surface $z=f(x)$ with expansion coefficients L_m a surface $z_1=a\cos\xi(2\pi/l_x)x$ with a period an integer times ξ smaller than the period of the function $f(x)$, for the coefficients of the surface $z+z_1$ we have the following expression:

$$L_m'=\sum_{k=-\infty}^{\infty}(-1)^kJ_k(\chi)L_{m-k\xi}.\quad(59)$$

Finally, consider a saw-tooth-shaped surface

$$z=f(x)=\begin{cases}a\left(1+\frac{4x}{l}\right), & -\frac{l}{2}\leqslant x\leqslant0\\ a\left(1-\frac{4x}{l}\right), & 0\leqslant x\leqslant\frac{l}{2}.\end{cases}\quad(60)$$

For the expansion coefficients of this surface we obtain

$$L_m'=\frac{i^{m+2}\chi\sin(m\pi/2+\chi)}{(m\pi/2)^2-\chi^2}.\quad(61)$$

For an analysis of transition radiation formulas let us consider the case of a one-dimensional sinusoidal surface (57). A parameter characterizing the roughness of the interface is the ratio of the sinusoid amplitude to the coherence length χ. For $\chi=0$ the formulas pass to the expressions for a plane interface $f(x,y)=0$ obtained by means of perturbation theory, since in this case only the zeroth order Bessel function is different from zero and hence no summation takes place. For small χ only the first terms of the series have noticeable values, since the Bessel function has an order of magnitude χ^m; for large χ the first terms are small.

From the expression for χ,

$$\chi=\frac{a}{\lambda}\frac{1}{\beta\sqrt{\varepsilon_0}\cos\psi}\times\left[1-\beta_x\sqrt{\varepsilon_0}\left(\sin\theta\cos\varphi-\frac{\lambda}{l_x}m\right)-\beta_z\sqrt{\varepsilon_0}\cos\theta\right],\quad(62)$$

it follows that a deep corrugation for small ψ gives the same radiation as a shallow one for large ψ.

For normal incidence of the electron on the target ($v_x = v_y = 0,\ v_z = v$) we have the following expression for the argument of the Bessel function:

$$\chi = \frac{a}{\lambda} \frac{1}{\beta\sqrt{\varepsilon_0}} (1 - \beta\sqrt{\varepsilon_0}\cos\theta). \tag{63}$$

This expression shows that if the condition for Vavilov-Cherenkov radiation is satisfied the zeroth order Bessel function takes the maximal value, i.e., no summation in formulas takes place.

ACKNOWLEDGMENTS

The author thanks M. L. Ter-Mikayelian for helpful discussions. This work was supported in part by the INTAS (Grant No. 97-30392).

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